Global Monopole in Asymptotically dS/AdS Spacetime

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(Dated: January 27, 2003)

Abstract : In this paper, we investigate the global monopole in asymptotically dS/Ads spacetime and find that the mass of the monopole in the asymptotically dS spacetime could be positive if the cosmological constant is greater than a critical value. This shows that the gravitational field of the global monopole could be attractive or repulsive depending on the value of the cosmological constant.

PACS numbers: 11.27.+d, 98.80.Cq

arXiv:hep-th/0210050 v1 5 Oct 2002

Various of kinds of topological defects could be produced by the phase transition in the early Universe and their existence has important implications in cosmology[1]. Global monopole, which has divergent mass in flat spacetime, is one of the most important above mentioned defects. The property of the global monopole in curved spacetime, or equivalently, its gravitational effects, was firstly studied by Barriola and Vilenkin[2]. When one considers the gravity, the linearly divergent mass of global monopole has an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin. Harari and Loustò[3], and Shi and Li[4] have shown that this small gravitational potential is actually repulsive. A new class of cold stars, addressed as D-stars(defect stars) have been proposed by Li et.al.[5, 6, 7]. One of the most important features of such stars, comparing to Q-stars, is that the theory has monopole solutions when the matter field is abscent, which makes the D-stars behave very differently from the Q-stars. The topological defects are also investigated in the Friedmann-Robertson-Walker spacetime[8].On the other hand, there has been a renewed interest in AdS spacetime due to the theoretical speculation of AdS/CFT correspondence, which state that string theory in anti-de Sitter space (usually with extra internal dimensions) is equivalent to the conformal field theory in one less dimension[9, 10]. Recently, the holographic duality between quantum gravity on de Sitter(dS) spacetime and a quantum field theory living on the past boundary of dS spacetime was proposed[11] and the vortices in dS spacetime was studied by Ghezelbash and Mann[12]. Many authors conjectured that the dS/CFT correspondence bear a lot of similarities with the Ads/CFT correspondence, although some interpretive issues remain. The monopole and dyon solution in gauge theories based on the various gauge group have been found[13]. However, in flat space there can not be static soliton solution in the pure Yang-Mills theory[14]. The presence of gravity can supply attractive force which binds non-Abelian gauge field into a soliton. The cosmological constant influence the behavior of the soliton solution significantly. In asymptotically Minkowski spacetime the electric components are forbidden in static solution[15]. If the spacetime includes the cosmological constant, forbidding the electric components of the non-Abelian gauge fields fail, thus allowing dyon solutions. A continuum of new dyon solutions in the Einstein-Yang-Mills theory in asymptotically AdS spacetime have been investigated[16], which are regular everywhere and specified with their mass, and non-Abelian electric and magnetic charges. Similarly, the presence of cosmological constant affects the behavior of the global monopole remarkably. If the spacetime is modified to include the positive cosmological constant, the gravitation field of global monopole can be attractive in contrast to the same problem in asymptotically Minkowski or AdS spacetime.

In this paper, we study the global monopole in asymptotically AdS/dS spacetime and show that the mass of the monopole might be positive in asymptotically dS spacetime if the cosmological constant is greater than a critical value.

The Lagrangian for the global monopole is

$$L = \frac{1}{2}\partial_\mu \phi^a \partial^\nu \phi^a - \frac{1}{4}\lambda^2(\phi^a\phi^a - \sigma_0)^2 \tag{1}$$

where ϕ^a is the triplet of Goldstone field and possesses a internal O(3) symmetry. When the symmetry breaks down

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to U(1), there will exist topological defects known as monopole. The configuration describing monopole solution is

$$\phi^a = \sigma_0 f(\rho)\frac{x^a}{\rho} \tag{2}$$

where $x^a x^a = \rho^2$ and $a = 1, 2, 3$.

When f approaches unity at infinity, we will have a monopole solution. The static spherically symmetric metric is

$$ds^2 = B(\rho)dt^2 - A(\rho)d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{3}$$

By introducing the dimensionless parameter $r = \sigma_0\rho$, we obtain the equations of motion for goldstone field as:

$$\frac{1}{A}f^{''} + [\frac{2}{Ar} + \frac{1}{2B}(\frac{B}{A})^{'}]f^{'} - \frac{2}{r^2}f - \lambda^2(f^2 - 1)f = 0 \tag{4}$$

where the prime denotes the derivative with respect to r.

In dS/AdS spacetime, the Einstein equation is

$$G_{\mu\nu} + \beta g_{\mu\nu} = \kappa T_{\mu\nu} \tag{5}$$

where β is the cosmological constant and $\kappa = 8\pi G$. dS and AdS spacetime correspond to the cases that β is positive and negative respectively. The Einstein equations in dS/AdS spacetime now are ready to written as:

$$-\frac{1}{A}(\frac{1}{r^2} - \frac{1}{r}\frac{A^{'}}{A}) + \frac{1}{r^2} = \epsilon^2 T^0_0 - \frac{\beta}{\sigma_0^2} \tag{6}$$

$$-\frac{1}{A}(\frac{1}{r^2} + \frac{1}{r}\frac{B^{'}}{B}) + \frac{1}{r^2} = \epsilon^2 T^1_1 - \frac{\beta}{\sigma_0^2} \tag{7}$$

where

$$T^0_0 = D + U + V \tag{8}$$
$$T^1_1 = D + U - V \tag{9}$$

are energy momentum tensors and

$$\begin{aligned} D &= \frac{f^2}{r^2} \\ U &= \frac{\lambda^2}{4}(f^2 - 1)^2 \\ V &= \frac{f^{'2}}{2A} \end{aligned} \tag{10}$$

and $\epsilon^2 = \kappa\sigma_0^2$ is a dimensionless parameter. Solving Eqs.(6) and (7), one can obtain

$$A^{-1}(r) = 1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}r^2 - \frac{2G\sigma_0 M_A(r)}{r} \tag{11}$$

$$B(r) = 1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}r^2 - \frac{2G\sigma_0 M_B(r)}{r} \tag{12}$$

where

$$M_A(r)=4\pi\sigma_0\exp[-\triangle(r)]\times\int_0^r dy\exp[\triangle(y)]\{f^2-1+y^2[U+(1-\epsilon^2+\frac{\beta}{3\sigma_0^2}y^2)f'^2]\}\tag{13}$$

and

$$M_B(r)=M_A(r)\exp[\widetilde{\triangle}(r)]+\frac{r(1-\epsilon^2+\frac{\beta}{3\sigma_0^2}r^2)}{2}\{1-\exp[\widetilde{\triangle}(r)]\}\tag{14}$$

In which

$$\triangle(r)=\frac{\epsilon^2}{2}\int_0^r dy(yf'^2)\tag{15}$$

and

$$\widetilde{\triangle}(r)=\epsilon^2\int_\infty^r dy(yf'^2)\tag{16}$$

Next, we discuss the behavior of these functions in asymptotically dS/AdS spacetime. A global monopole solution f should approaches unity when $r\gg 1$. If this convergence is fast enough then $M_A(r)$ and $M_B(r)$ will also quickly converge to finite values. Therefore, we can find the asymptotic expansions:

$$f(r)=1-\frac{3\sigma_0^2}{\beta+3\lambda^2\sigma_0^2}\frac{1}{r^2}-\frac{9[2\beta\epsilon^2\sigma_0^4+3(2\epsilon^2-3)\lambda^2\sigma_0^6]}{2(2\beta-3\lambda^2\sigma_0^2)(\beta+3\lambda^2\sigma_0^2)^2}\frac{1}{r^4}+O(\frac{1}{r^6})\tag{17}$$

$$M_A(r)=M_A(\beta,\epsilon^2)+\frac{4\pi\sigma_0}{r}+O(\frac{1}{r^3})\tag{18}$$

$$M_B(r)=M_A(r)(1-\frac{\epsilon^2}{r^4})+\frac{4\pi\sigma_0(1-\epsilon^2)}{r^3}+O(\frac{1}{r^7})\tag{19}$$

where $M_A(\beta,\epsilon^2)\equiv\lim_{r\to\infty}M_A(r)$, which is a function dependent on β and ϵ^2. The dependence on ϵ of asymptotic behavior is quite weak for the global monopole solution. However, the asymptotic behavior is evidently dependent of the parameters β, σ_0 and λ. The Eqs.(17)-(19) in the limit of small cosmological constant reduce to the well known ones in flat spacetime. The solution Eqs.(11)-(12) induce a deficit angle in asymptotically dS/AdS spacetime. Numerical calculation for $f(r)$ show that its shape is quite insensitive to ϵ in the range $0\le\epsilon\le 1$ not only asymptotically, but also close to the origin. We also find that an increasing positive cosmological constant tends to make a thicker monopole solution and a decreasing negative cosmological constant tends to make a thinner monopole solution.

In the following, We present a numerical analysis to the system and the results are shown in Fig.1.

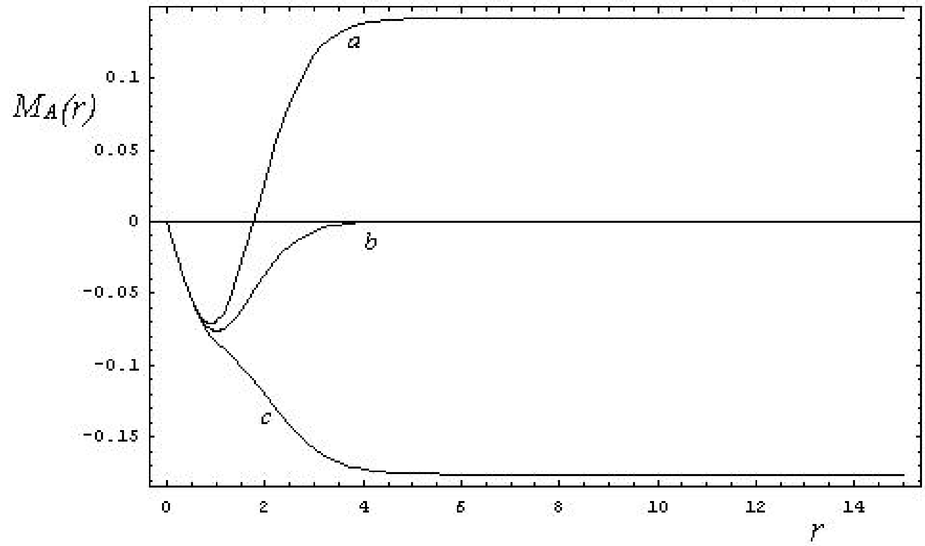

Fig1. The plot of mass as a function of r. Here we choose $\lambda = 1$, $\sigma_0 = 0.01$, $G = 1$. Curve (a), (b) and (c) are plotted when $\beta = 0.0010, 0.0003$ and -0.0005 respectively.

From Fig.1, one can find that the mass decrease to a negative asymptotic value when r approaches infinity in AdS spacetime. But in dS spacetime, the mass will be positive if the cosmological constant is large enough. The critical value for the cosmological constant is 0.0003 in our set-up. The asymptotic mass for the above Curve (a), (b) and (c) are 0.1415, 0.0000 and -0.1760 respectively. It is clear that the presence of cosmological constant affects the behavior of the global monopole significantly. If the spacetime is modified to include a positive cosmological constant, the gravitational field of global monopole can be attractive in contrast to the same problem in asymptotically flat or AdS spacetime.

Finally, We want to discuss why the tiny mass of global monopole is defined as the limiting value of the function $M_A(r)$ given by Eq.(13). The standard definition of the ADM mass is different: It is defined by the $g_{rr}^{-1} = 1 - \frac{2GM(r)}{r} - \frac{\lambda}{3}r^2$, where λ is the cosmological constant. The mass, M, is then the limiting value of $M(r)$, and it is always positive. This agrees with the well-known positive mass theorem for the dS/AdS space firstly proved by [17]. However, in the case of global monopole, $M(r)$ is linearly divergent for large r, which leads to a deficit solid angle plus a residual effect of gravitational field. When a test particle moves in the gravitational field of the global monopole, the repulsive or attractive nature of the residual gravitational effect can be perceived by this particle[3, 4]. That is, the standard definition of the mass gives in the case of a global monopole a linearly divergent expression plus a constant term. Now, it turns out that the divergent term does not produce any gravitational effect on the matter interacting with the monopole, and the whole interaction is entirely determined by the subleading finite term. This is why it is customary to subtract the divergent term from the definition of mass, since it is the resulting finite 'effective' mass that determines the gravitational interaction with the monopole. Furthermore, the attractive or repulsive property of the residual gravitational field is determined by the positiveness or negativeness of $M_A(r)$. For the relation between the Positive Mass Conjecture and global monopole, one may refer to Ref.[18], in which Cvetiĉ and Soleng pointed out that "In Ref[19], a Positive Mass Conjecture was formulated saying that there is no singularity free solution of Einstein's field equations with matter sources(not including the vacuum) obeying the weak energy condition equations for which an exterior observer can see a negative mass object. A global monopole Ref[2, 3, 4] would appear to be a counter example, but in this case the Goldstone fields extended to infinity, which means that these objects are extended sources and all observers must be inside the system". Therefore, although the effective mass M_A is negative under certain circumstance, it will not contradict the Positive Mass Conjecture[19] and the Positive Mass theorem in dS/AdS spacetime[17].

When one study the motion of test particles around a global monopole, it is an excellent approximation to take $M_A(r)$ as the constant $M_A(\beta, \epsilon^2)$ since the effective mass approaches its asymptotic value very quickly. Apart from the academic interest in the global monopole configuration, the D-stars[5, 6, 7] seem to make it relevant to astronomical situation. Work on the generalization of D-stars to asymptotically ds/Ads spacetime is in preparation.

ACKNOWLEDGMENTS

This work was partially supported by National Nature Science Foundation of China under Grant No. 19875016, National Doctor Foundation of China under Grant No. 1999025110, and Foundation of Shanghai Development for Science and Technology under Grant No. 01JC1435.

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